Mathematics Department Math 205A
List of Main Topics

1. Measure Zero, Lebesgue's Theorem on the Riemann integral

2. sigma-algebras in abstract space, Borel sets in topological space.

3. Lebesgue outer measure in $\mathbb{R}^n$; Caratheodory's criterion for measurability.

4. General outer measures, and Caratheodory's criterion for measurability.

5. Abstract measure spaces; relation to outer measures.

6. In any measure space, proof that the limit of the measures of an increasing sequence of sets is the same as the measure of the union of the sets. Analogous result for decreasing sequences in case at least one has finite measure.

7. Caratheodory condition for Borel regularity of an outer measure in a metric space; example of the Hausdorff measures on $\mathbb{R}^n$

8. Measurable functions, simple functions, uniform approximation from below of non-negative bounded measurable functions by non-negative simple functions and pointwise approximation from below of arbitrary extended real valued non-negative measurable function by non-negative simple functions. Egoroff's theorem.

9. Lebesgue integral of non-negative measurable functions f in abstract measure space as the sup of integral of simple function, over all non-negative simple functions which are $\leq f$ everywhere; note these simple functions may be positive on a set of infinite measure.

10. Lebesgue integrable functions in abstract measure space.

11. Convergence theorems (Fatous, Monotone convergence theorem, Dominated convergence theorem)

12. Product measures and Fubini's theorem.

13. Five times covering in $\mathbb{R}^n$, and proof of the corresponding Vitali lemma.

14. Differentiation theory for monotone, BV and AC functions.

15. Differentiation theory for Lebesgue measure in $\mathbb{R}^n$ using Vitali.

16. Borel regular and Radon measures, and discussion of extent to which 15 extends to other Radon measures. Lusin for Radon measures in locally compact separable metric spaces.

17. L^p-spaces: definition, Minkowski inequality, Holder inequality, and the L^p norm. Completeness of L^p

18. Hahn decomposition, Jordan decomposition, Radon-Nikodym theorem, Lebesgue decomposition theorem

19. Riesz representation theorem for L^p, $1 \leq p$, p finite.

20. Riesz representation for linear functionals on the continuous functions with compact support in locally compact Hausdorff spaces.